Is the IMF a *probability density distribution function* or is star formation a self-regulated process?

Observatoire astronomique de Strasbourg
25th of September 2015

Pavel Kroupa

Helmholtz-Institut fuer Strahlen und Kernphysik (HISKP)

Helmholtz Institute for Radiation and Nuclear Physics

c/o Argelander-Institut für Astronomie

University of Bonn

the IMF

$$dN = \xi(m)\, dm, \quad \text{the number of stars in } m,\ m+dm$$

. . . as derived from **detailed star-count analyses**
(Kroupa, Tout & Gilmore 1991; 1992, 1993; Kroupa et al 2013)

The Canonical IMF

(m is in units of $M_\odot$)

$$\xi_{BD}(m) = \frac{k}{3}\left(\frac{m}{0.07}\right)^{-0.3\pm0.4} \quad , \quad 0.01 < m \lesssim 0.15,$$

$$\xi_{star}(m) = k\begin{cases}\left(\frac{m}{0.07}\right)^{-1.3\pm0.3} & , \quad 0.07 < m \le 0.5, \\ \left[\left(\frac{0.5}{0.07}\right)^{-1.3\pm0.3}\right]\left(\frac{m}{0.5}\right)^{-2.3\pm0.36} & , \quad 0.5 < m \le 150.\end{cases} \qquad (4.55)$$

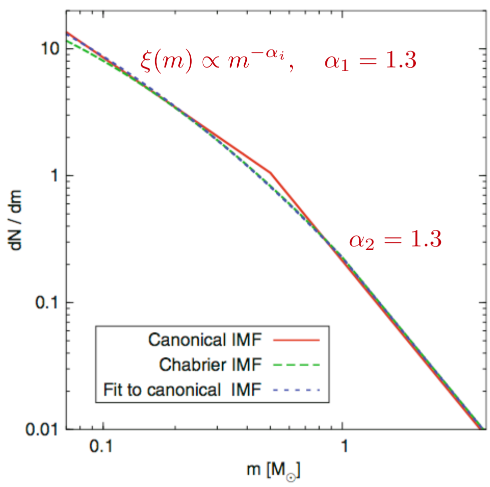
The Log-normal Canonical IMF

(m is in units of $M_\odot$)

$$\xi_{BD}(m) = k\, k_{BD}\left(\frac{m}{0.07}\right)^{-0.3\pm0.4} \quad , \quad 0.01 < m \lesssim 0.15,$$

$$\xi_{star}(m) = k\begin{cases}\frac{1}{m}\exp\left[-\frac{(lm - lm_c)^2}{2\sigma_{lm}^2}\right] & , \quad 0.07 < m \le 1.0, \\ A\left(\frac{m}{1.0}\right)^{-2.3\pm0.36} & , \quad 1.0 < m \le 150.\end{cases}$$

$\sigma_{lm} = 0.75, \quad A = 0.244, \quad k_{BD} = 4.46$ (not Chabrier !)



The
IMF
as a
scale-invariant
probability density
distribution function

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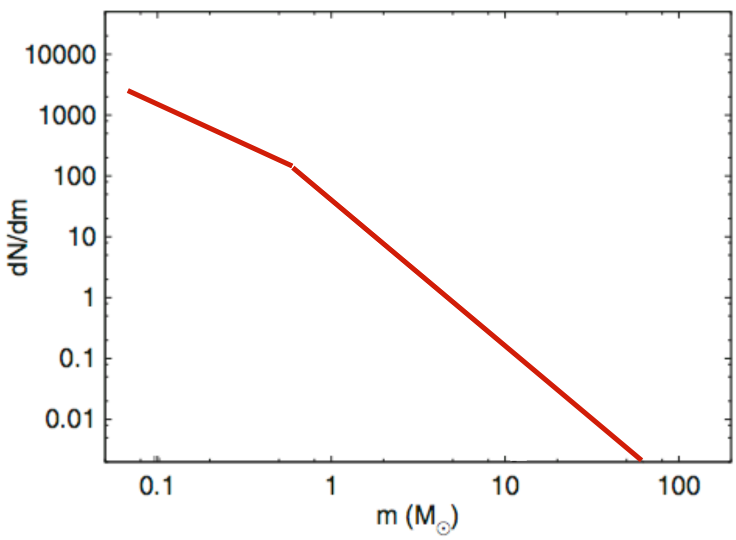
The IMF as a scale-invariant *probability density distribution function*

==> stochastically sampled stellar populations when discretised :



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The IMF as a scale-invariant *probability density distribution function*

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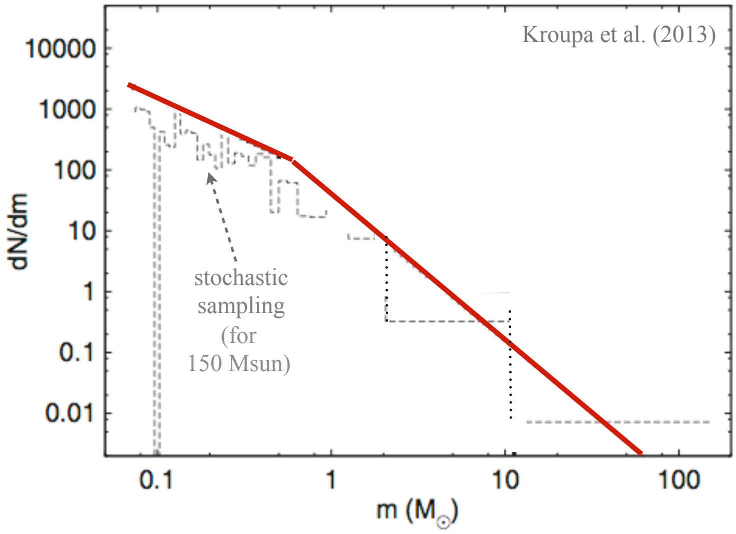


==> stochastic variations in the shape of the IMF from case to case.

Elmegreen - many papers
Bastian et al. 2010

or ***fractions of stars*** in low-mass galaxies and low-mass star clusters when analytical description ==> this is ***unphysical*** but often employed

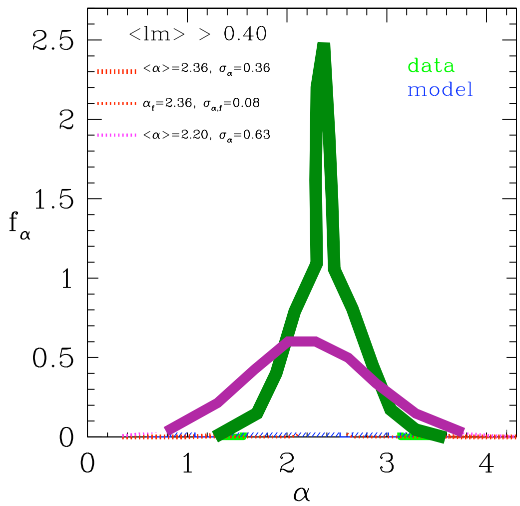
Tests : is the IMF a scale-invariant pdf ?

Is the standard assumption (invariant probabilistic IMF) consistent with observations **?**

1. upper mass of stars is limited (to about 150 Msun)
==> i.e ∃ physical constraints on the IMF

(Weidner & Kroupa 2004; Figer 2005; Oey & Clarke 2005; Koen 2006; Maiz Appelaniz et al. 207)

2. observed variations of IMF shapes much smaller than in theory



$\xi(m) \propto m^{-\alpha}, \; m > 2.5\, M_\odot$

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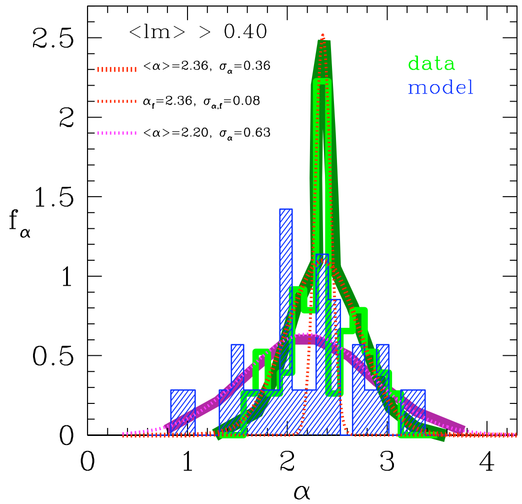

$\xi(m) \propto m^{-\alpha},\ m > 2.5\,M_\odot$

1. No assymmetries and sharp Salpeter/Massey peak.

2. Model *worse* than data **!?**

. . . but model has no measurement uncertainties

3. observed systematic change of galaxy-wide IMFs with SFR

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **415,** 1647–1662 (2011) doi:10.1111/j.1365-2966.2011.18800.x

Galaxy and Mass Assembly (GAMA): the star formation rate dependence of the stellar initial mass function

M. L. P. Gunawardhana,[1,2,3★] A. M. Hopkins,[2★] R. G. Sharp,[2] S. Brough,[2] E. Taylor,[3] J. Bland-Hawthorn,[3] C. Maraston,[4] R. J. Tuffs,[5] C. C. Popescu,[6] D. Wijesinghe,[3]

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ABSTRACT

The stellar initial mass function (IMF) describes the distribution in stellar masses produced from a burst of star formation. For more than 50 yr, the implicit assumption underpinning most areas of research involving the IMF has been that it is universal, regardless of time and environment. We measure the high-mass IMF slope for a sample of low-to-moderate redshift galaxies from the Galaxy and Mass Assembly survey. The large range in luminosities and galaxy masses of the sample permits the exploration of underlying IMF dependencies. A strong IMF–star formation rate dependency is discovered, which shows that highly star-forming galaxies form proportionally more massive stars (they have IMFs with flatter power-law slopes) than galaxies with low star formation rates. This has a significant impact on a wide variety of galaxy evolution studies, all of which rely on assumptions about the slope of the IMF. Our result is supported by, and provides an explanation for, the results of numerous recent explorations suggesting a variation of or evolution in the IMF.

1660 *M. L. P. Gunawardhana et al.*

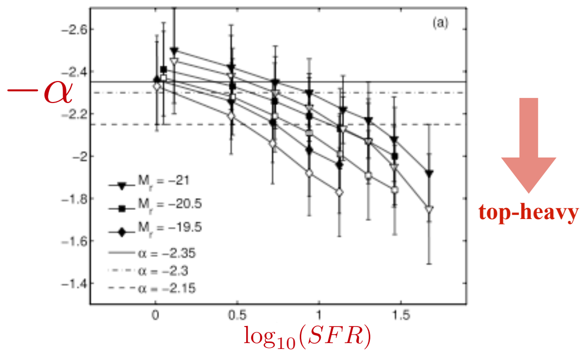

Figure 13. The best-fitting IMF slope for each of the (a) SFR (Fig. 6), (b) specific SFR (Fig. 11) and (c) SFR surface density (Fig. 12) sub-groups of the three volume-limited samples used in this study. The filled symbols denote results when using the Calzetti (2001)/Cardelli et al. (1989) dust corrections and open symbols represent the Fischera & Dopita (2005) dust corrections. The solid horizontal line indicates a Salpeter slope, the dot–dashed line indicates a Kroupa (2001) high-mass slope of $\alpha = -2.3$ and the dashed line dictates the Baldry & Glazebrook (2003) IMF slope.

$$\xi(m) \propto m^{-\alpha}$$

Very comparable results by

Hoversten E. A., Glazebrook K., 2008, ApJ, 675, 163

Meurer G. R. et al., 2009, ApJ, 695, 765

Lee, J. C. et al., 2009, ApJ, 706, 599

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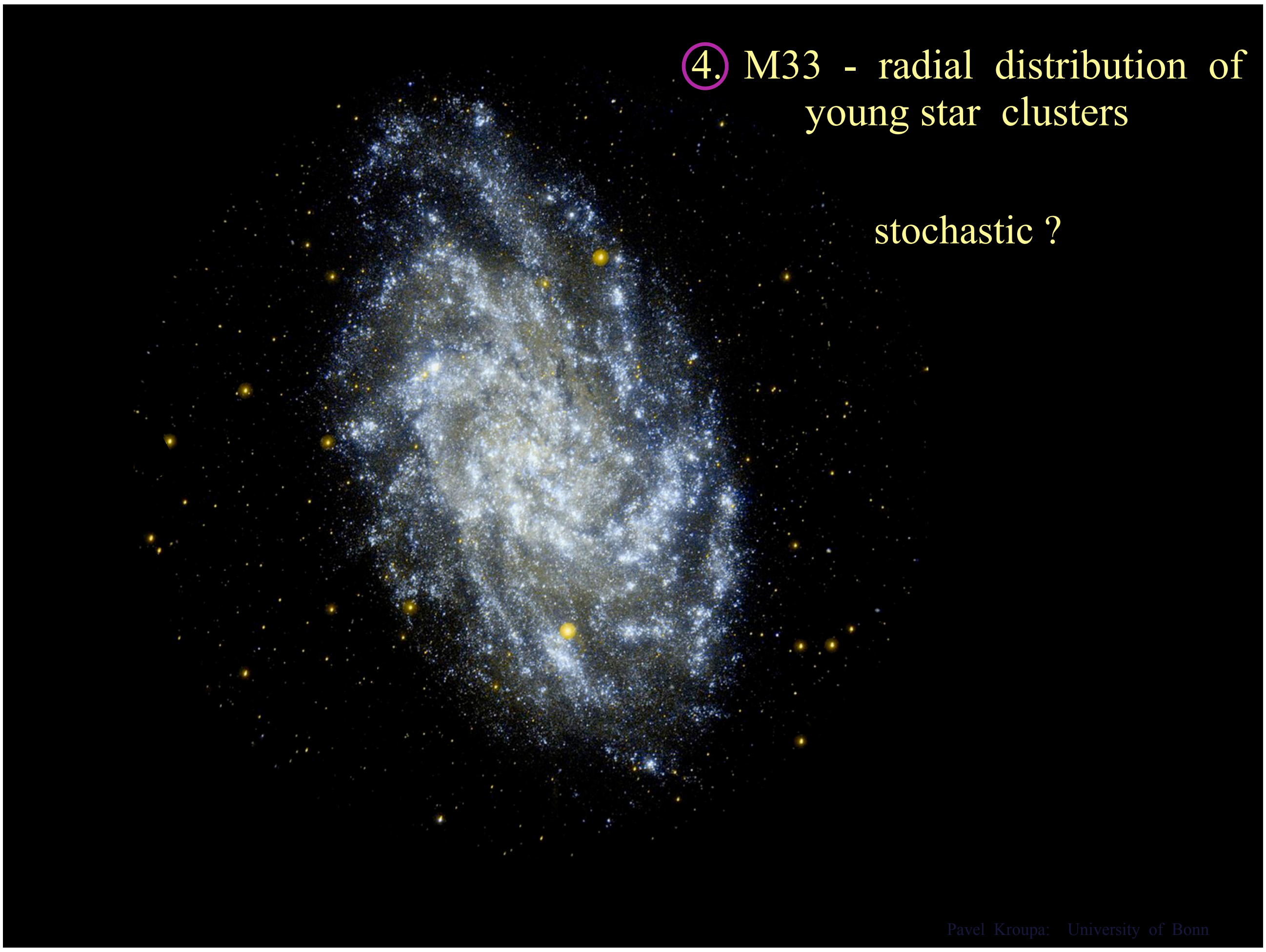

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **435**, 2604–2609 (2013) doi:10.1093/mnras/stt1474
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The galactocentric radius dependent upper mass limit of young star clusters: stochastic star formation ruled out

Jan Pflamm-Altenburg,[1]★ Rosa A. González-Lópezlira[1,2]★ and Pavel Kroupa[1]★
[1]*Argelander-Institut für Astronomie (AIfA), University of Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany*
[2]*Centro de Radioastronomía y Astrofísica, UNAM, Campus Morelia, Michoacán C.P. 58089, Mexico*

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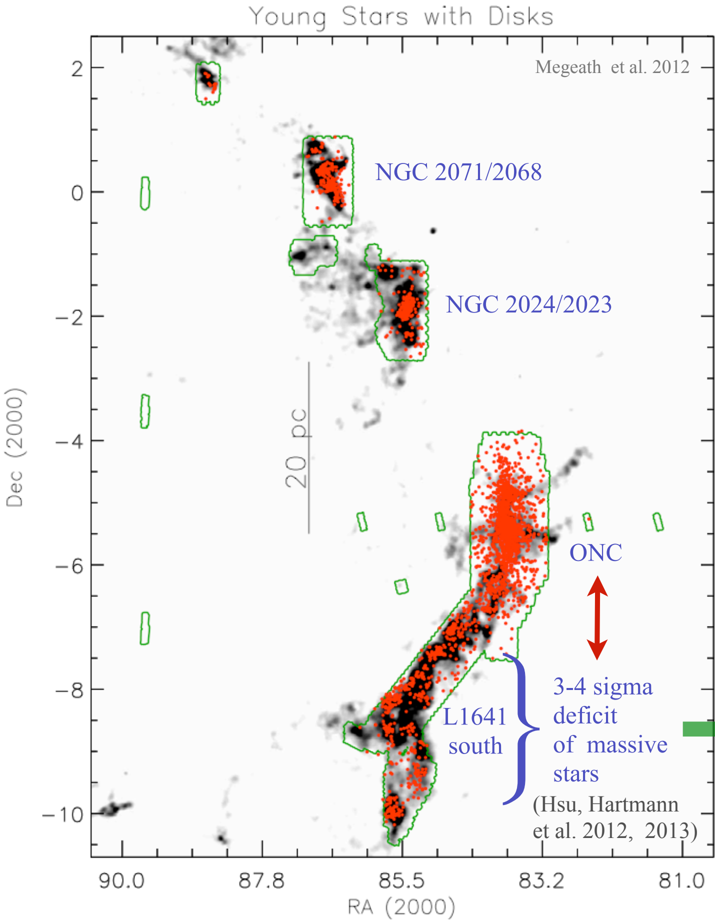
ABSTRACT

It is widely accepted that the distribution function of the masses of young star clusters is universal and can be purely interpreted as a probability density distribution function with a constant upper mass limit. As a result of this picture the masses of the most massive objects are exclusively determined by the size of the sample. Here we show, with very high confidence, that the masses of the most massive young star clusters in M33 decrease with increasing galactocentric radius in contradiction to the expectations from a model of a randomly sampled constant cluster mass function with a constant upper mass limit. Pure stochastic star formation is thereby ruled out. We use this example to elucidate how naive analysis of data can lead to unphysical conclusions.

Key words: stars: formation – galaxies: individual: M33 – galaxies: star clusters: general.

5 CONCLUSION

We have found that the formation of very massive star clusters is increasingly suppressed with increasing galactocentric radius in M33. We have ruled out with extremely high significance that this is the result of a size-of-sample effect, where a constant and environment independent ICMF is populated entirely randomly and environmental effects can be neglected.



5. Lack of O stars

Many small / low-mass groups or clusters do not yield the same IMF as one massive cluster

stochastic IMF in each group is ruled out.

1. + 2. + 3. + 4. + 5. →

Real young populations
are *not stochastic ensembles*
from invariant distr. functions

~~the IMF / star cluster MF **an invariant probability density distribution function**~~

The

IMF

as an

optimally sampled

probability density

distribution function

What is an optimally sampled distribution function ?

Given the *mass reservoir* in stellar mass M_{ecl},
starting with the most massive star,
select the next most massive such that
M_{ecl} is distributed over the distribution function
without Poisson scatter.

Kroupa et al. 2013
Schulz, Pflamm-Altenburg & Kroupa 2015

The above ansatz can be extended to a discretized optimal distribution of stellar masses: Given the mass, M_{ecl}, of the population, the following sequence of individual stellar masses yields a distribution function which exactly follows $\xi(m)$,

$$m_{i+1} = \int_{m_{i+1}}^{m_i} m\, \xi(m) dm, \quad m_L \le m_{i+1} < m_i, \quad m_1 \equiv m_{max}. \tag{4.9}$$

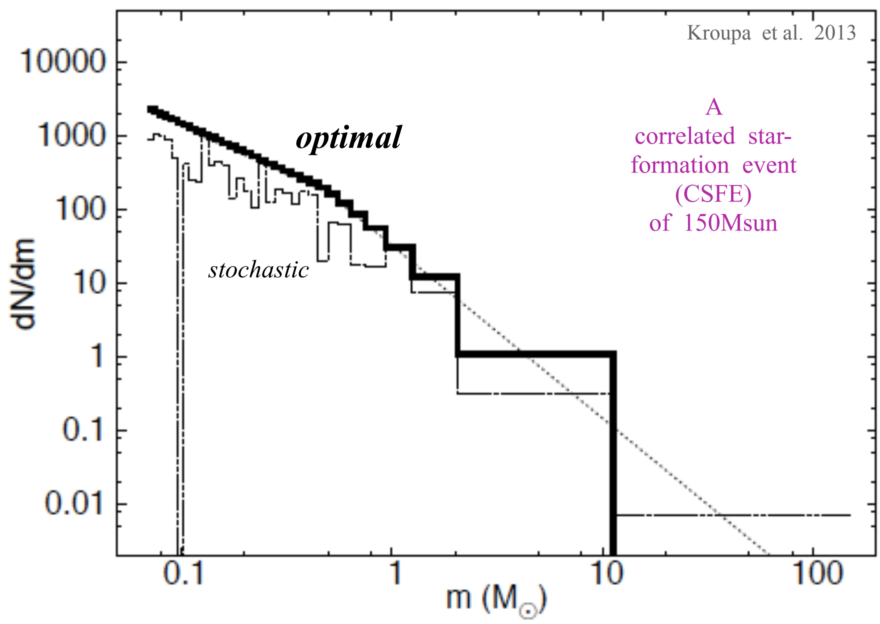
The normalization and the most massive star in the sequence are set by the following two equations:

$$1 = \int_{m_{max}}^{m_{max*}} \xi(m)\, dm, \tag{4.10}$$

with

$$M_{ecl}(m_{max}) - m_{max} = \int_{m_L}^{m_{max}} m\, \xi(m)\, dm \tag{4.11}$$

as the closing condition. These two equations need to be solved iteratively. An excellent approx-



the IMF as an **optimally-sampled distribution function**

(The Orion case suggests that the most massive star may depend on cluster mass.)

Consider : from above in each *correlated star-formation event* (**CSFE** $\equiv$ embedded cluster) there is *one most massive star* **:**

$$1 = \int_{m_{\rm max}}^{m_{\rm max*}} \xi(m)\, dm$$
$$M_{\rm ecl} = \int_{m_{\rm l}}^{m_{\rm max}} m\, \xi(m)\, dm$$

$\Rightarrow$ $m_{\rm max} = {\rm fn}(M_{\rm ecl})$

an mmax -- Mecl relation

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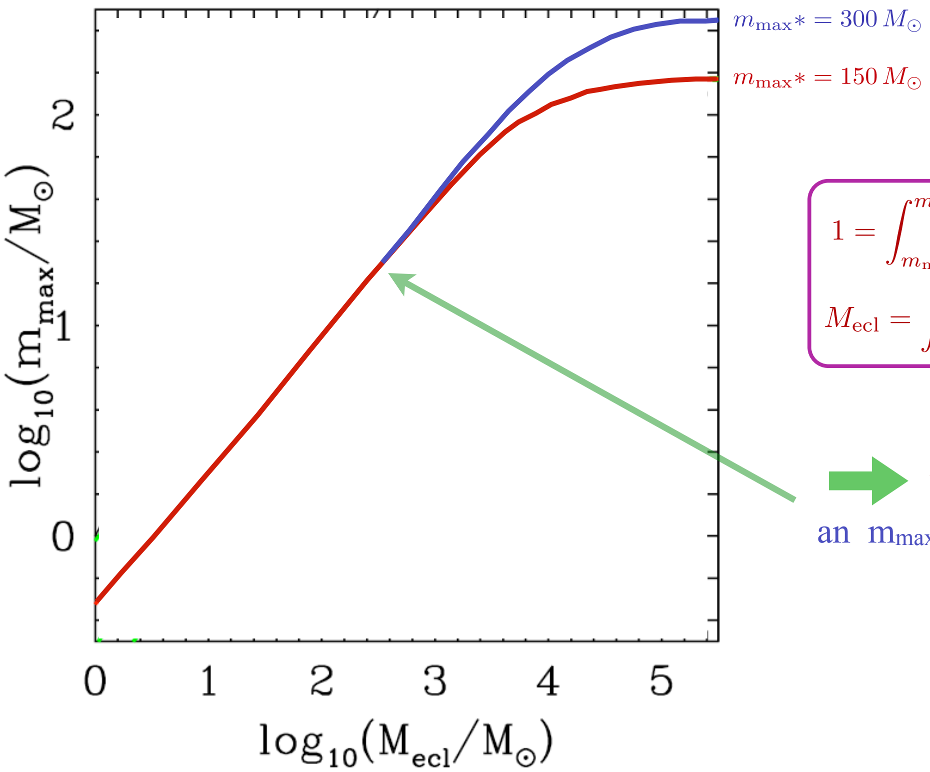

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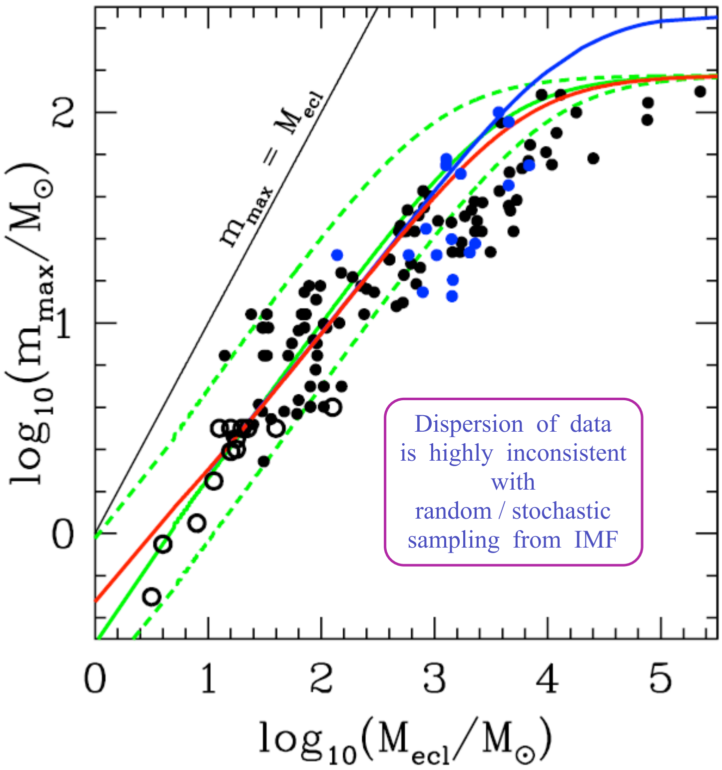


$m_{max*} = 300\, M_\odot$

Weidner & Kroupa 2005, 2006;
Weidner et al. 2010, 2013; Kroupa et al. 2013;
Kirk & Myers, 2010, 2012; Hsu, Hartmann et al. 2012, 2013

$m_{max*} = 150\, M_\odot$

$$1 = \int_{m_{max}}^{m_{max*}} \xi(m)\, dm$$

$$M_{ecl} = \int_{m_l}^{m_{max}} m\, \xi(m)\, dm$$

$\Rightarrow$ $m_{max} = \mathrm{fn}(M_{ecl})$

an mmax -- Mecl relation

A RESULT :
the scatter in the data
is mostly due to being from
measurement uncertainties,
i.e. ***intrinsic scatter may be very small***.

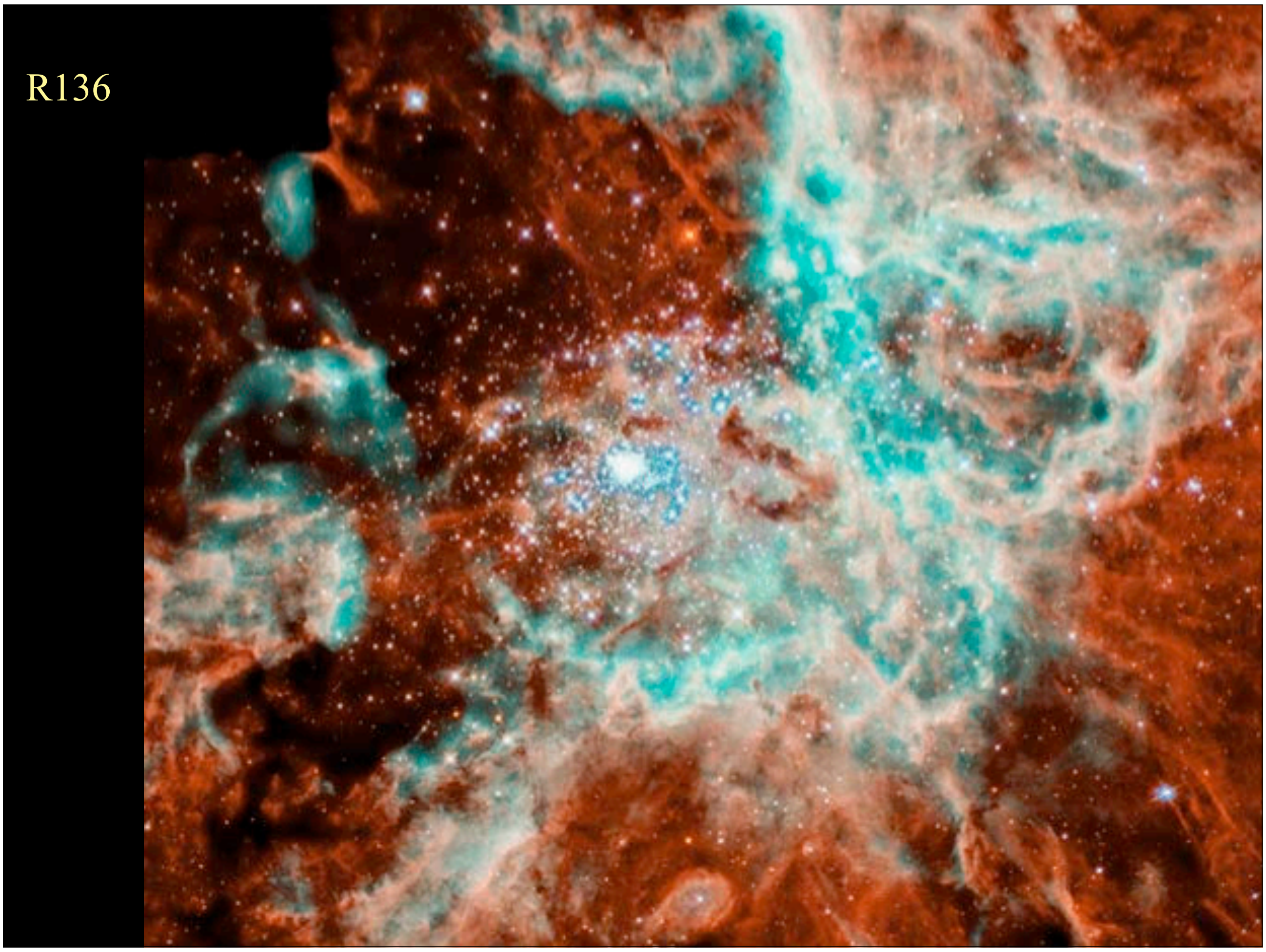
$\Downarrow$

The IMF can only be a probability distribution function,
if *sampling from it is close to optimal* **:**

the IMF as an **optimally sampled density distribution**

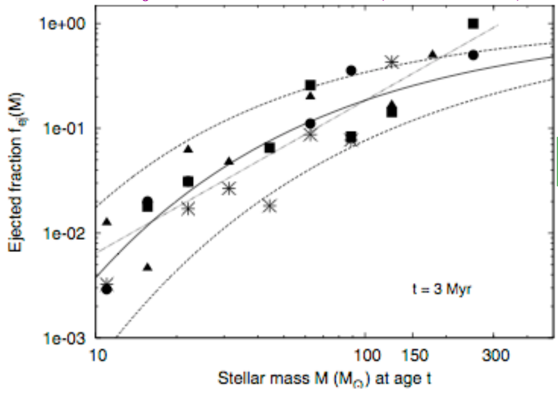
and by implication, star-formation is largely **self-regulated**

Evidence for a top-heavy IMF

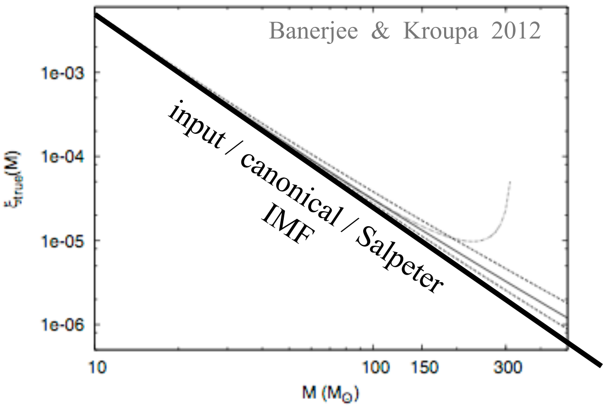


Nbody model of R136

Ejection fraction = fn(stellar mass)



Corrected IMF



IMF becomes top heavy with increasing density

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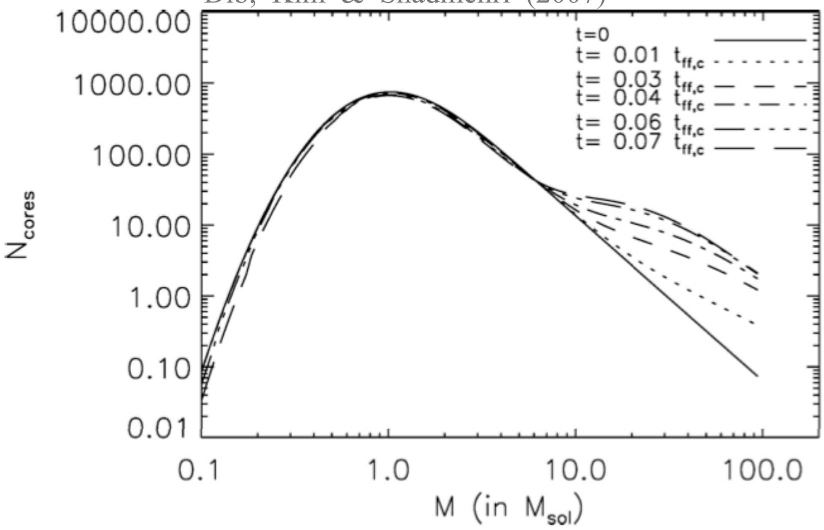
IMF becomes top heavy with increasing density :

Dib, Kim & Shadmehri (2007)



Models of coalescing and collapsing cloud cores in a dense proto cluster.

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***Globular clusters* :** deficit of low-mass stars increases with decreasing concentration

→ disagrees with dynamical evolution

→ correlate energy needed to expell residual gas with number of OB stars required. Marks et al. 2012

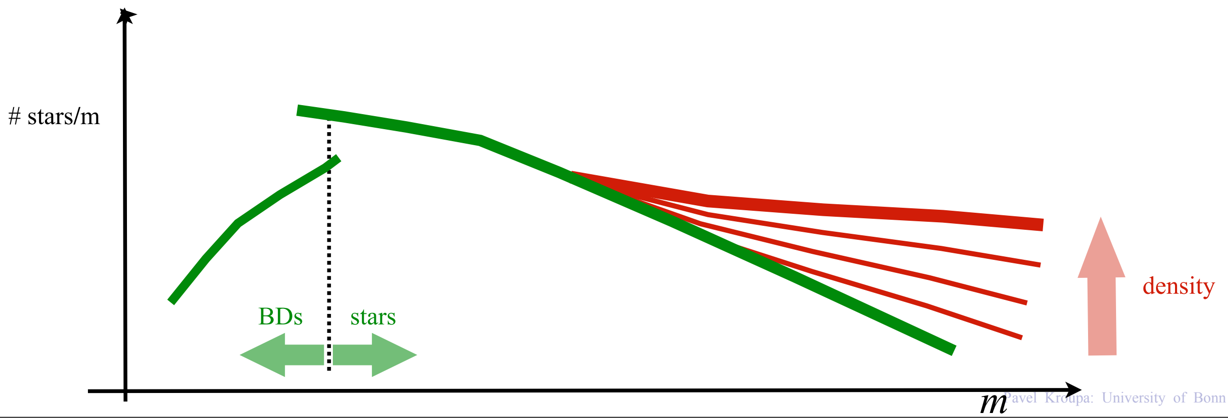
***UCDs* :** higher dynamical M/L ratios

→ cannot be exotic dark matter => top-heavy IMF Dabringhausen et al. 2009

***UCDs* :** larger fraction of X-ray sources than expected

→ no explanation other than many remnants => top-heavy IMF Dabringhausen et al. 2012

What this implies :



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Globular clusters : deficit of low-mass star ... tration

disagree...

correlate ... gas with
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Marks et al. 2012

UCDs : higher dynamical M/L ratios

cannot ... IMF

Dabringhausen et al. 2009

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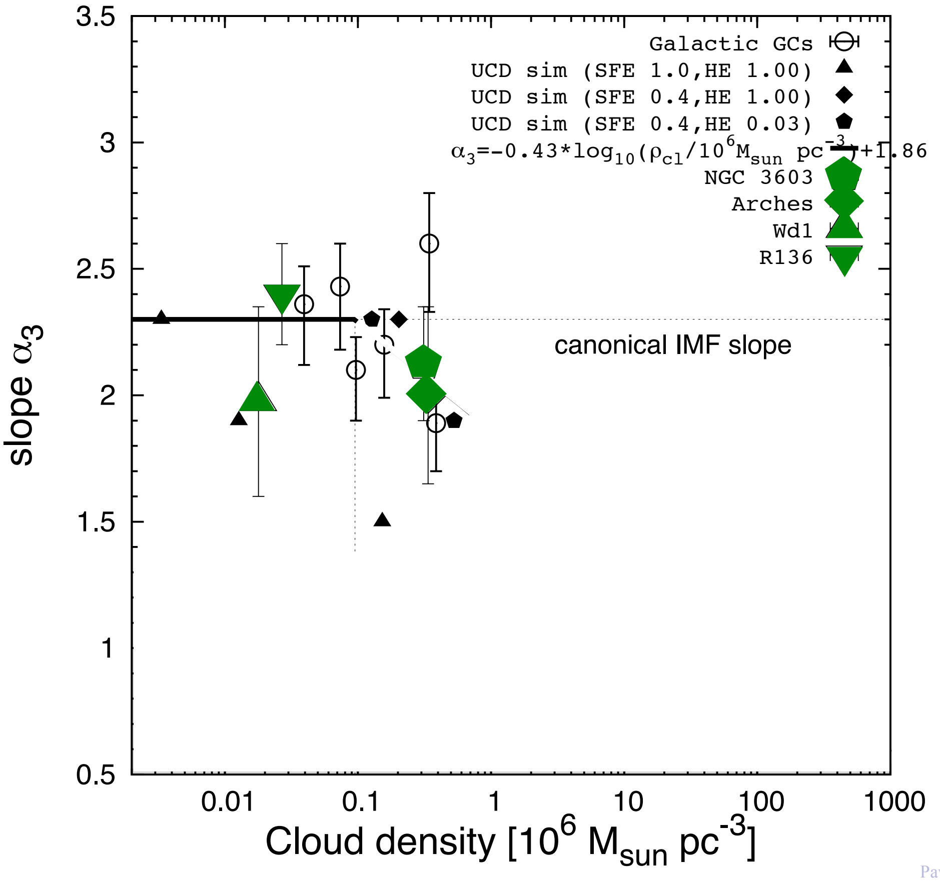
no exp... => top-heavy IMF

Dabringhausen et al. 2012

What this implies :

Joerg Dabringhausen

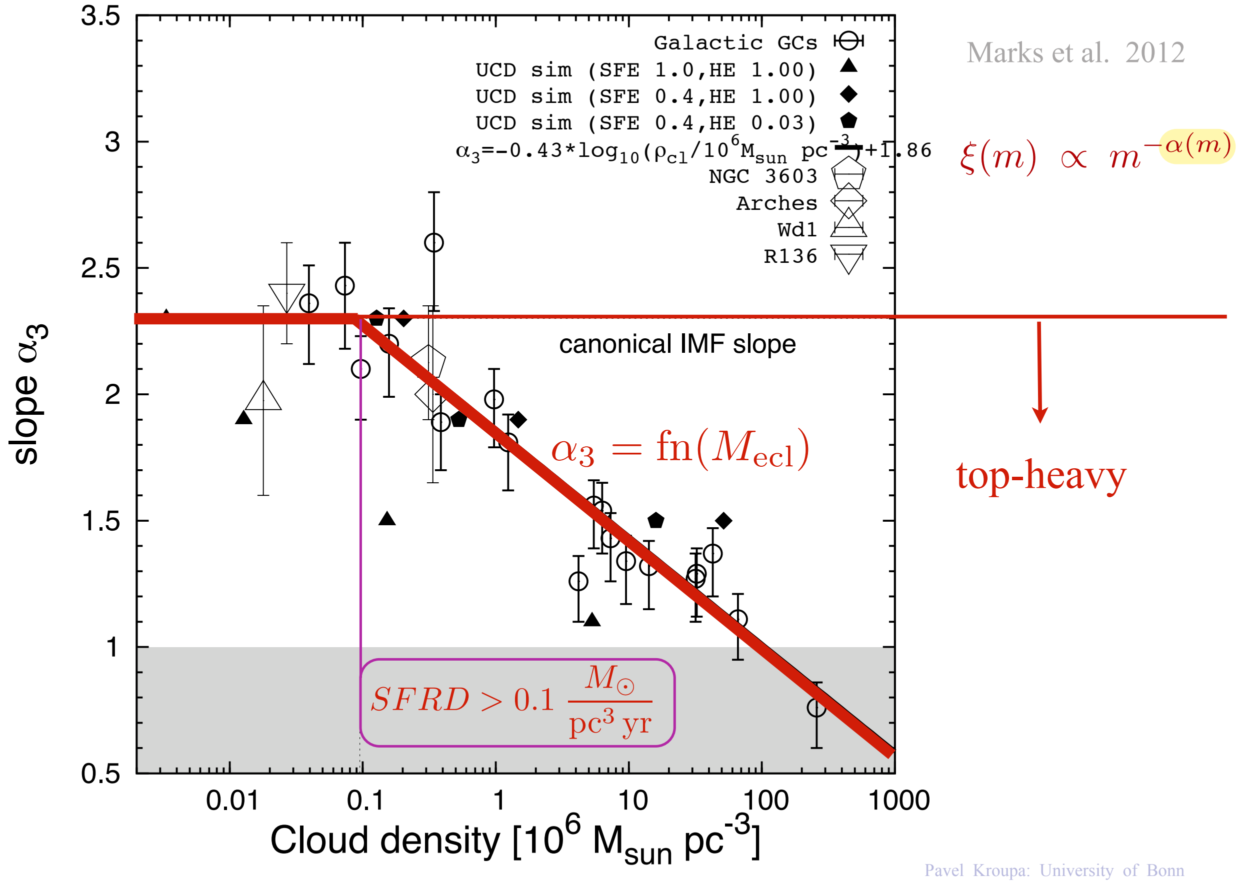
Top-heavy IMF in extreme-density environments :



Marks et al. 2012

$$\xi(m) \propto m^{-\alpha(m)}$$

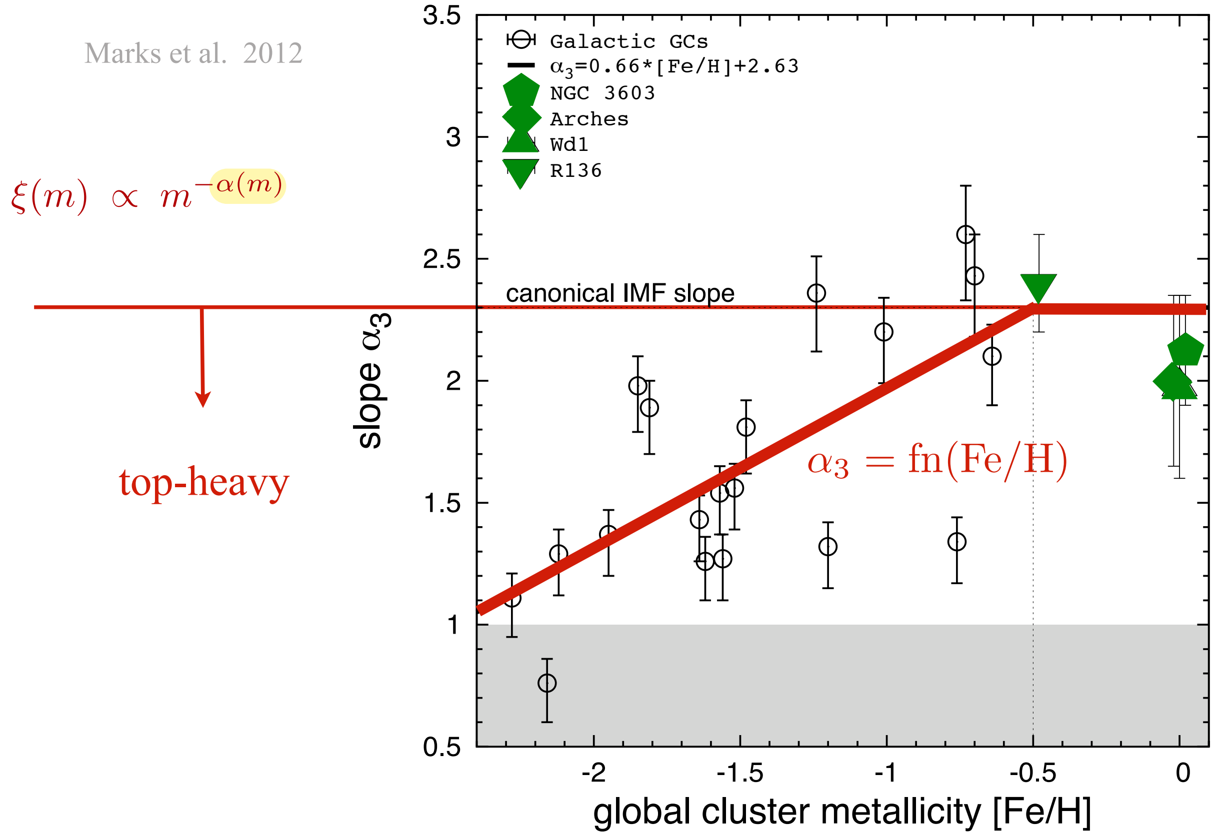
Top-heavy IMF in extreme-density environments :



Marks et al. 2012

Top-heavy IMF in extreme-density environments :

Marks et al. 2012



Top-heavy IMF in extreme-density environments :

THE STELLAR IMF DEPENDENCE ON DENSITY AND METALLICITY: Resolved stellar populations show an invariant IMF (Eq. 55), but for $SFRD \gtrsim 0.1\, M_\odot/(\mathrm{yr\ pc}^3)$ the IMF becomes top-heavy, as inferred from deep observations of GCs. The dependence of α_3 on cluster-forming cloud density, ρ, (stars plus gas) and metallicity, [Fe/H],can be parametrised as

$$\alpha_3 = \alpha_2, \qquad m > 1\, M_\odot \quad \wedge \quad x < -0.89,$$
$$\alpha_3 = -0.41 \times x + 1.94, \qquad m > 1\, M_\odot \quad \wedge \quad x \geq -0.89,$$
$$x = -0.14\, [\mathrm{Fe/H}] + 0.99 \log_{10} \left(\rho / \left(10^6\, M_\odot\, \mathrm{pc}^{-3} \right) \right) . \qquad (65)$$

Marks et al. 2012
Kroupa et al. 2013 (arXiv:1112.3340)
Recchi & Kroupa 2014 (arXiv:1411.0318)

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Using for $m < 1\, M_\odot$

$$\alpha_{1,2} = \alpha_{1/2\mathrm{c}} + 0.5 \left[\frac{\mathrm{Fe}}{\mathrm{H}} \right]$$

estimated from resolved MW populations
(Kroupa 2001)

where the canonical solar-abundance values are
$\alpha_{1\mathrm{c}} = 1.3 \quad 0.07 < m/M_\odot < 0.5$
$\alpha_{2\mathrm{c}} = 2.3 \quad 0.5 < m/M_\odot < 1.3$

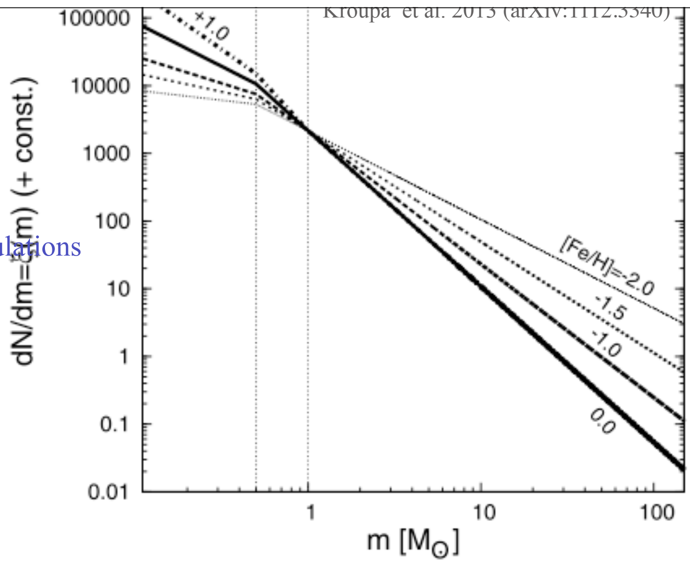

Figure 5. Suggested shape of the stellar IMF for different metallicities, [Fe/H] (not taking into account the density dependence of the IMF). The IMFs are scaled such that their values agree at $m = 1\,\mathrm{M}_\odot$. Above $1\,\mathrm{M}_\odot$ the IMF slope is determined by the present work (Fig. 4, equation 11). Below $1\,\mathrm{M}_\odot$ the parametrization is determined by Kroupa (2001, equation 12), whose results suggest tentative evidence that more metal-rich environments produce relatively more low-mass stars. Note that only the metallicity dependence is shown, but not the dependence on mass (Fig. 2) or density (Fig. 3).

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Thus there is good and independent quantifiable evidence for the IMF becoming top-heavy with increasing density and decreasing metallicity.

Note: extraction of this evidence requires *understanding* the data and of *the dynamical evolution* of the CSFEs.

IMF = IMF(Z, $SFRD$)

Z=metallicity, $SFRD$=star-formation rate density

From correlated star formation events (CSFEs or embedded clusters) to galaxies

Correlated star formation events (CSFEs)

The stellar population of an individual CSFE is of stellar mass $M_{\rm ecl}$

Assuming star formation takes place in CSFEs, the stellar population from an ensemble of CSFEs can be computed, if the *distribution of CSFEs* is known.

CSFE-mass distribution :

$$\xi_{\rm ECMF}(M_{\rm ecl}) = k\, M_{\rm ecl}^{-\beta}; \quad \beta \approx 2$$

(Lada & Lada 2003)

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Correlated star formation events building up a galaxy

The total mass in stars formed in a galaxy over time δt is $M_{\rm tot} = SFR \times \delta t$

But $$M_{\rm tot} = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}} \xi_{\rm ecl}(M_{\rm ecl})\, M_{\rm ecl}\, dM_{\rm ecl}$$

For $M_{\rm ecl,min} = 5\, M_\odot$ and with $$1 = \int_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}} \xi_{\rm ecl}(M_{\rm ecl})\, dM_{\rm ecl}$$

where $M_{\rm ecl,max*} \approx 10^7\, M_\odot$

Thus $M_{\rm ecl,max} = {\rm fn}(SFR)$

What is *delta t* ?

The galaxy-wide time-scale of transforming the ISM via molecular clouds into a new stellar population (Egusa et al. 2004; 2009).

Disappearance of large molecular clouds around young star clusters (Leisawitz 1989).

→ $\delta t \approx 10$ Myr

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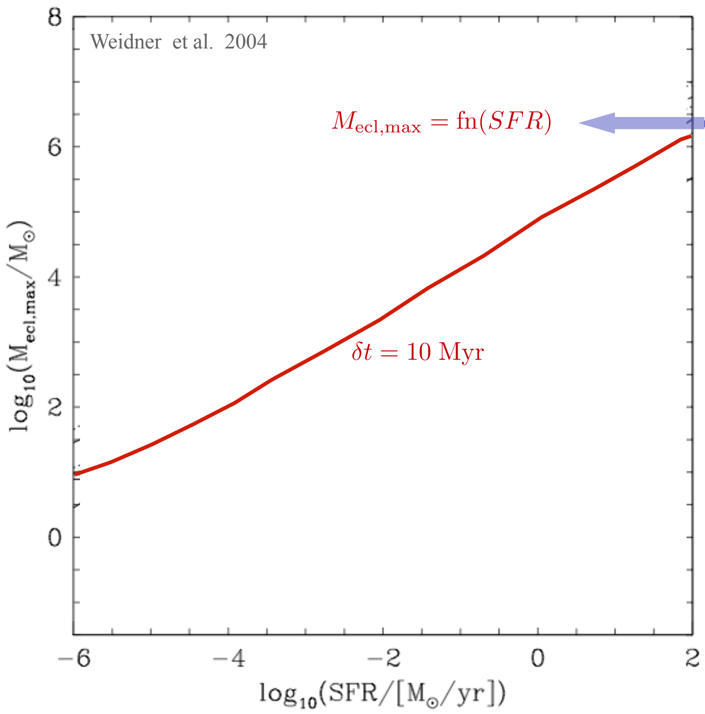


$$M_{\rm tot} = SFR \times \delta t$$

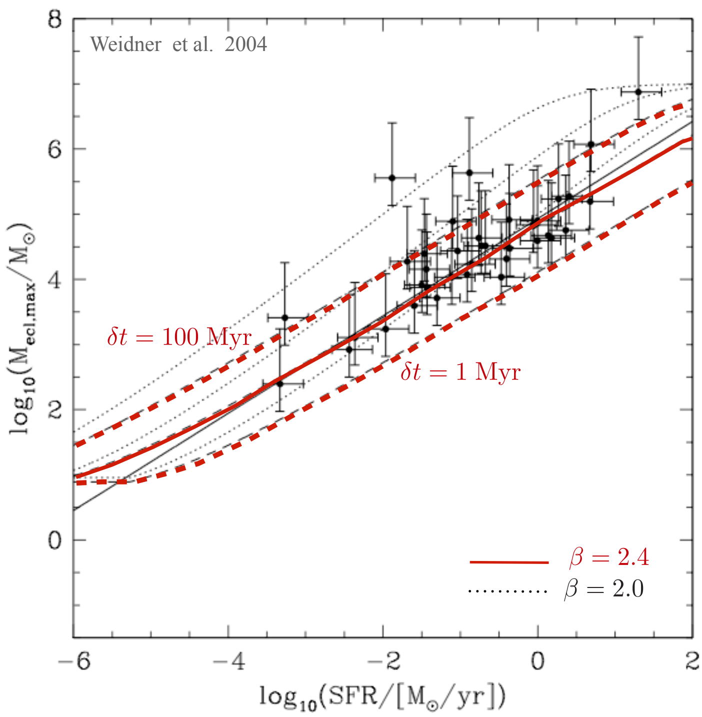
$$M_{\rm tot} = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}} \xi_{\rm ecl}(M_{\rm ecl})\, M_{\rm ecl}\, dM_{\rm ecl}$$

$$1 = \int_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}} \xi_{\rm ecl}(M_{\rm ecl})\, dM_{\rm ecl}$$

$\delta t = 10$ Myr

$M_{\rm ecl,min} = 5\ M_\odot$

$M_{\rm ecl,max*} \approx 10^7\ M_\odot$



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$\delta t = 10$ Myr

$M_{\rm ecl,min} = 5\ M_\odot$

$M_{\rm ecl,max*} \approx 10^7\ M_\odot$

Composite Stellar Populations

The **I**ntegrated **G**alactic **IMF** now follows from

$$\xi_{\rm IGIMF}(m,t) = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}(SFR(t))} \xi(m \le m_{\rm max}(M_{\rm ecl}))\ \xi_{\rm ecl}(M_{\rm ecl})\, dM_{\rm ecl}$$

Kroupa & Weidner (2003); Weidner & Kroupa (2005, 2006)

Vanbeveren (1982)

adding-up all IMFs
in all SCFEs!
The LEGO principle

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IGIMF = ∑ of IMFs (in all CSFEs/ embedded clusters)

Natural explanation of the
mass-metallicity relation
of galaxies
and many other problems in
understanding galaxies.

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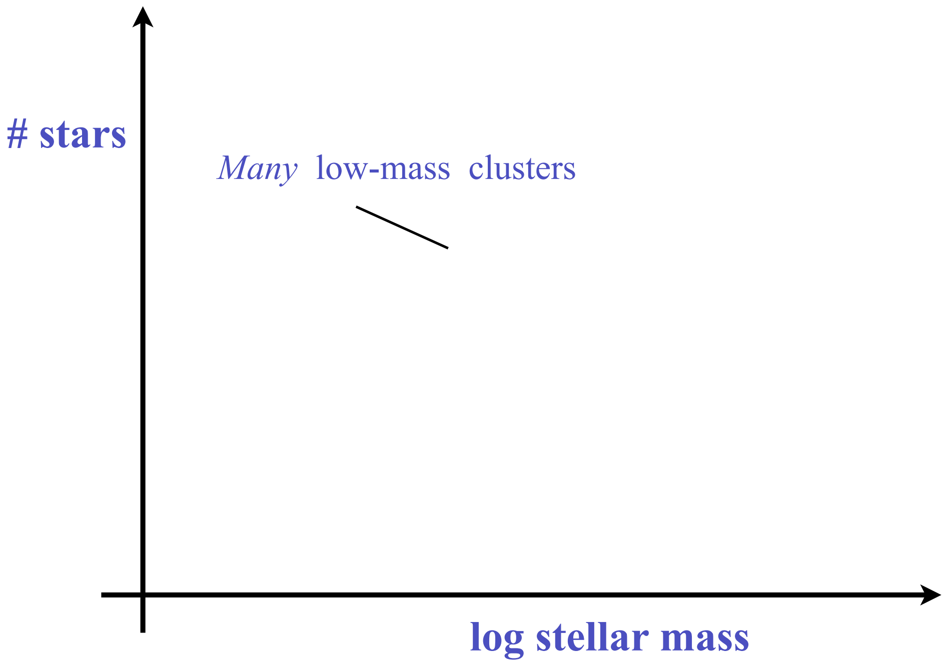
Why is the IGIMF different to the IMF?



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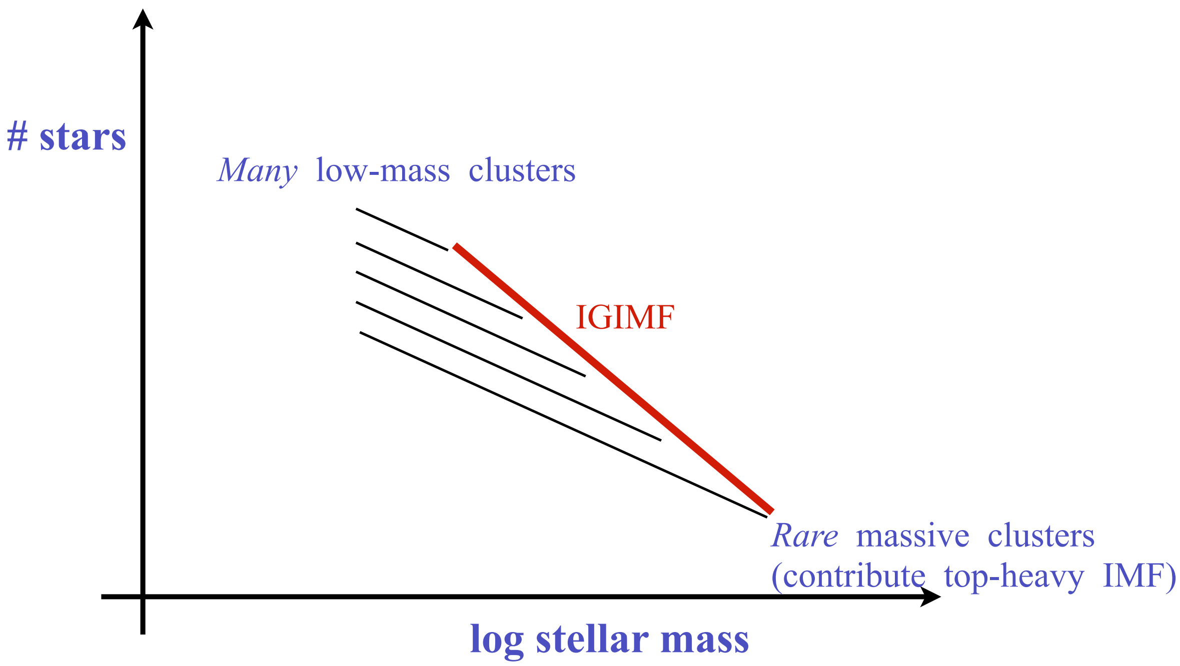

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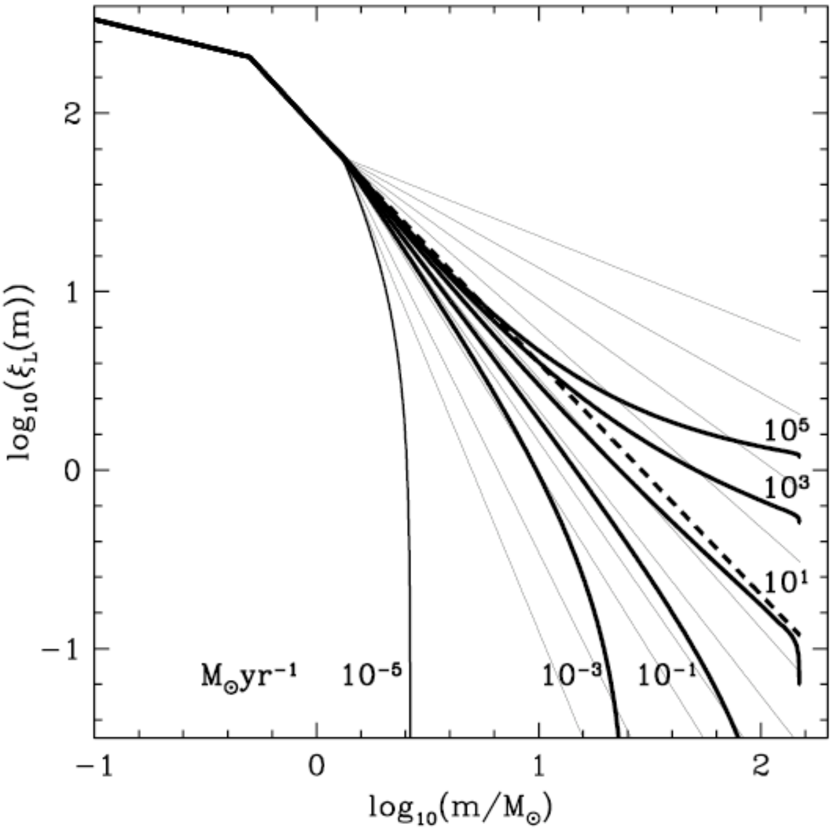
Weidner et al. 2013; Kroupa et al. 2013



The IGIMF for galaxies with different SFRs

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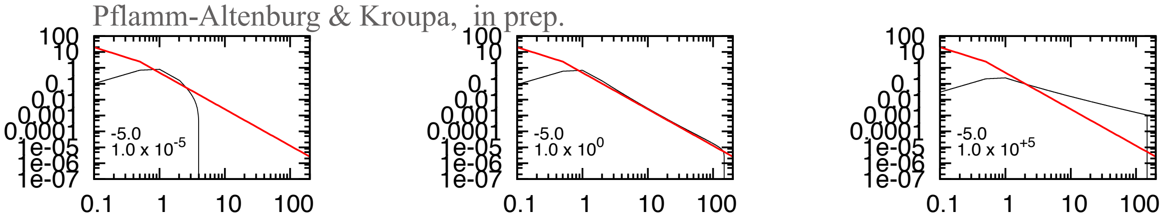
The IGIMF for galaxies with different SFRs & [Fe/H]

Pflamm-Altenburg & Kroupa, in prep.



Pavel Kroupa: University of Bonn

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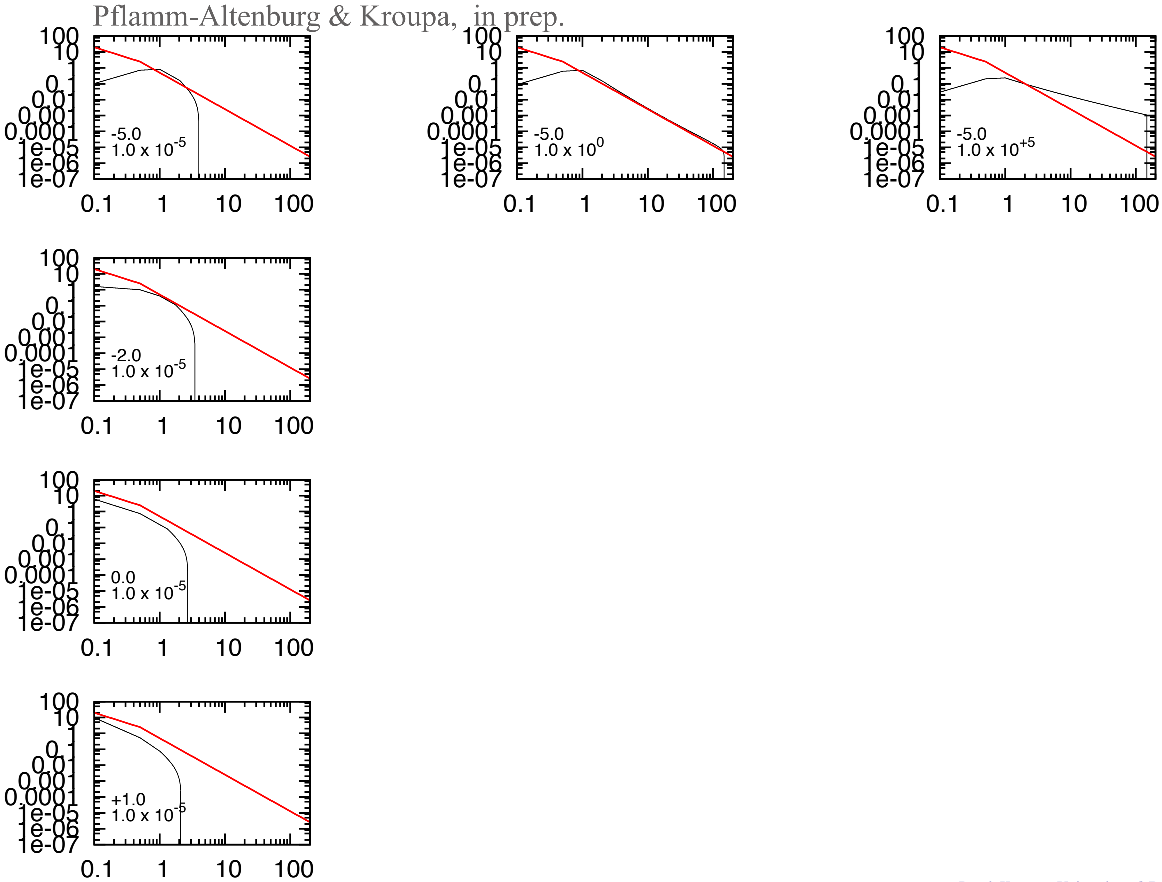

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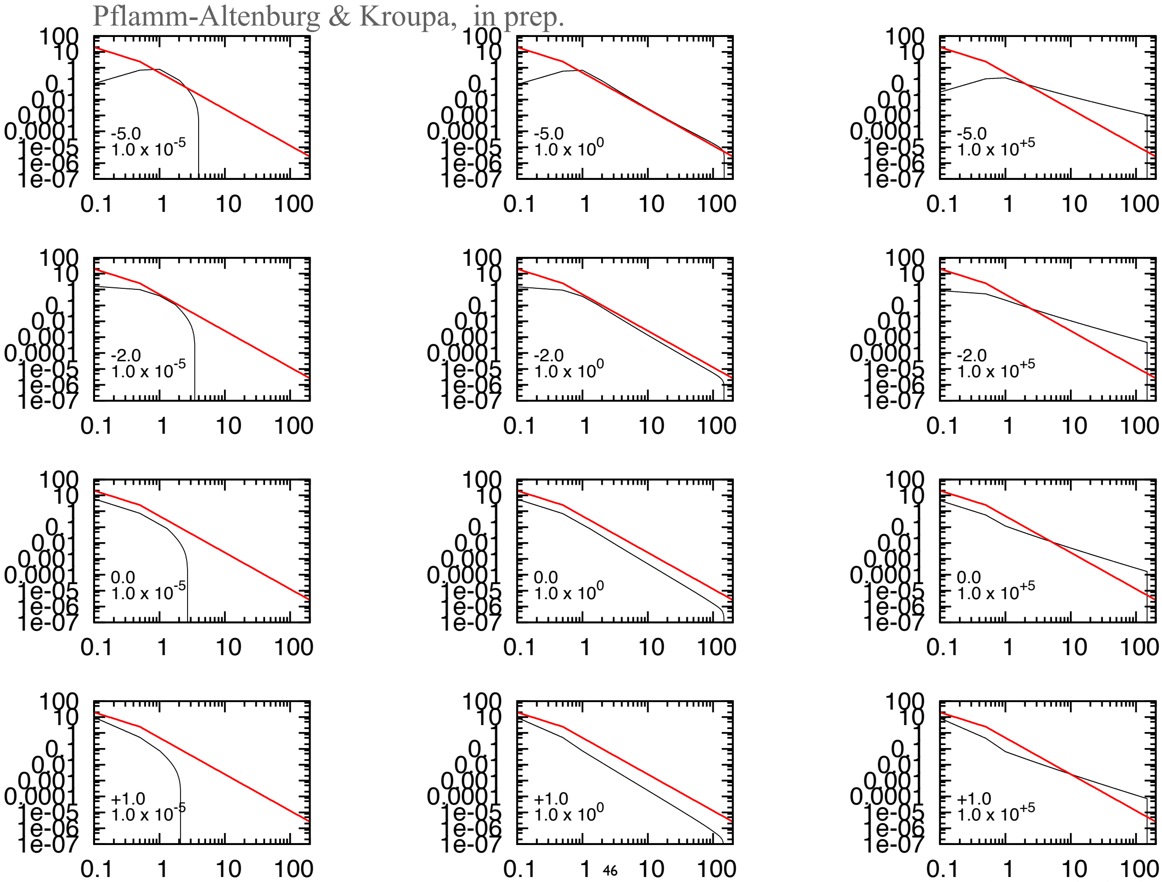
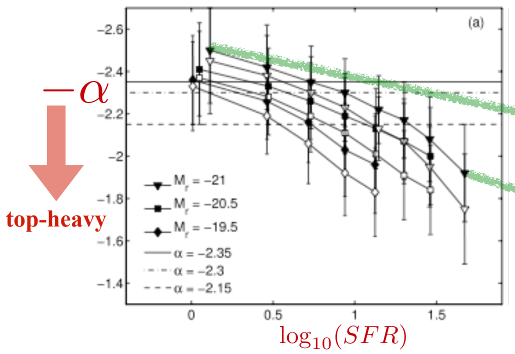
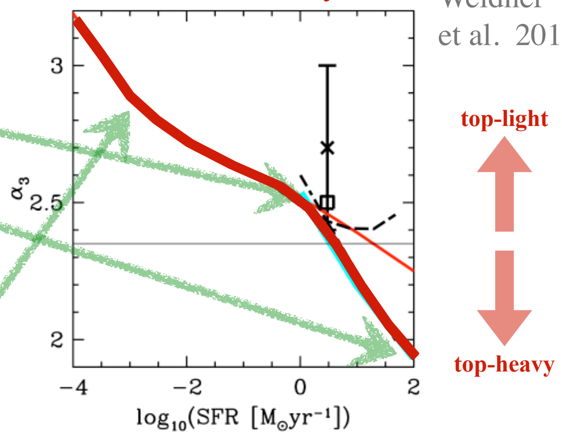


1660 *M. L. P. Gunawardhana et al.* *2011*



IGIMF theory

Weidner et al. 2013



Very comparable / consistent results by

Hoversten E. A., Glazebrook K., 2008, ApJ, 675, 163

Meurer G. R. et al., 2009, ApJ, 695, 765

Lee, J. C. et al., 2009, ApJ, 706, 599

E galaxies formed with top-heavy IMFs

confirming Matteucci (1994) !

but see recent work by Vazdekis

Conclusions

- The *stellar IMF* is found to be *surprisingly invariant* up to a critical star-formation rate density (SFRD) on a <pc scale
- Above this critical SFRD [GCs + UCDs] ==> evidence for top-heavy IMF in starbursts $SFRD > 0.1 \frac{M_\odot}{\mathrm{pc}^3\,\mathrm{yr}}$
- IMF : *not* a probabilistic / stochastic distribution function
- IMF : appears to be closer to an *optimally* sampled distribution function (**i.e. star-formation is feedback regulated**)
- **Superposition principle :** Sum over all CSFEs (=embedded clusters) **:** very powerful approach to quantify properties of freshly hatched stellar populations in galaxies **:**

  IGIMF, binary properties, thick disks
- Pioneering application of this concept in self-consistent full-scale hydrodynamic simulations of galaxies by *Ploeckinger et al. (2014)*

Both,
Sverre Aarseth &
Chris Tout from
Cambridge,
have influenced this
work profoundly.